2020

RMMSC

Rocky Mountain Society for Music Theory

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Putting the Math in Math Rock

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Slides: https://tinyurl.com/Chiu-PMMR-Slides
Handout/References: https://tinyurl.com/Chiu-PMMR
*The **talk with animations** is located here:* **https://www.youtube.com/watch?v=hbpi5giwkV0**

Introduction

In the late 20th century, progressive rock and alternative punk coalesced into a new genre: "math rock." To start, let's put ourselves in the mathy ambiance by listening to a bit of "Chinchilla" by the band TTNG—try to find the meter as you listen. (**https://tinyurl.com/TTNGChinchilla**) For the record, the verse uses groupings of 11 + 13. As you may have just heard, math rock contains "extensive use of asymmetrical…time signatures" (Cateforis 2002, 244) and frequent shifts in meter—the obsessive counting involved in its discourse is how the "math rock" nomenclature was formed. Coincidentally, the "cyclical repetition of ostinati" (Osborn 2010, 43) in math rock is aptly modeled by a mathematical equation: the discrete Fourier transform (henceforth DFT). In fact, the DFT *assumes* cyclic structures, giving it an analytical bias uniquely suited to study math rock's rhythms. I not only suggest that the DFT is appropriate for the genre, but that it represents a cognitively-informed model of meter—modeling aptitude for metric entrainment.

I've split the talk into four parts: 1) a short introduction to the math rock genre; 2) an explanation of the DFT, connecting the abstract mathematics to music cognition; 3) a description of my methodology; and 4) an analysis of excerpts from three math rock songs. In doing so, I aim to add to the growing discourse of rhythm and meter by examining an underrepresented musical style through a new theoretical lens.

So… what is Math Rock and where did it come from?

Part I. Math-rock

While any stylist genealogy is complex, we know for sure that the earliest math bands were inspired by progressive-rock bands like King Crimson and Yes (see Figure 1, below). Math bands like Don Caballero (Pittsburgh PA) and Chavez (New York, NY) are some of the genre's first flagships and represent an earlier style more akin to post-rock. Many recent math-rock bands, however, are adopting the math-emo nomenclature and even attract a similar fanbase. The term "emo" comes from the D.C. hardcore scene in the mid 80s—short for "emotional hardcore." As scholars have defined it, one defining characteristic of the contemporary emo genre is its timbral *twinkle* in the guitars (Howie 2020), but as the emo style gains popularity and becomes more widespread, its sound becomes less ubiquitous (Eberhart 2016).[1] If you're interested in exploring math-rock/emo style more, I've attached a short spotify playlist you can listen to on your handout. For now, I'll examine bands which lean towards the emo side of math rock. I'll be examining 3 math-rock songs: "Never Meant" by the indisputably most influential fourth-wave math-emo band: American Football; "Pool" by an all femme Japanese band Tricot; and "Cat Fantastic" by the UK band TTNG.

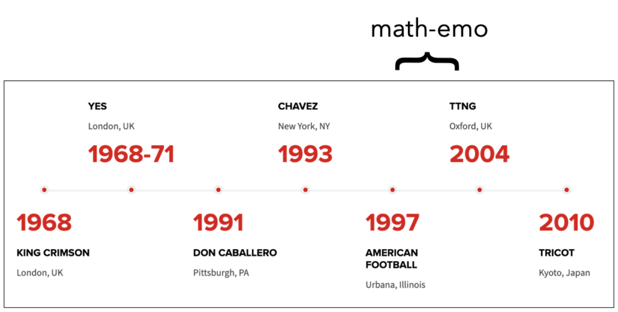

Figure 1. Math rock timeline

[1] The twinkly sound is a relatively natural electric guitar sound with added compression pedal. Some artists differentiate their styles with different amounts of overdrive. Yvette Young uses a lot while Tiny Moving Parts reserve overdrive for particular moments in the piece.

We'll be using the DFT to examine rhythms in these songs, so first I'll explain what the DFT is and how it works.

Part II. The discrete Fourier transform

The DFT converts a signal, or input, into its sinusoidal components—I won't discuss the math here, but if you're interested, your online handout has a color-coded equation. Essentially, the DFT's task is to break down and retrieve information from a signal, and ours, as interpreters, is to relate these individual elements to the whole.

While the algorithm for the equation has been around since the early 19th century, in music theory, its introduction started with David Lewin. David Lewin was the first to propose using the DFT in music theory in 1959. However, the DFT remained untouched by music theorists until Lewin himself returned to it in 2001. Ian Quinn and Clifton Callender picked up his torch and were the first to explore the DFT extensively. Since then, the DFT has been applied and discussed in various contexts, as shown on the slide behind me. Only recently has the DFT been implemented in the rhythmic domain. (Milne, Bulger, Herff 2017; Chiu 2018; Yust forthcoming MTS).

In order to calculate the DFT, we require some form of input. Because this paper works with symbolic music, its input will be an array of numbers representing *positions in time* (see Figure 2). To determine the number of non-trivial *components* resulting from the DFT, divide the length of the array by 2. So if we divide 4/4 into eighth notes, there are 8 positions and therefore 4 relevant components.

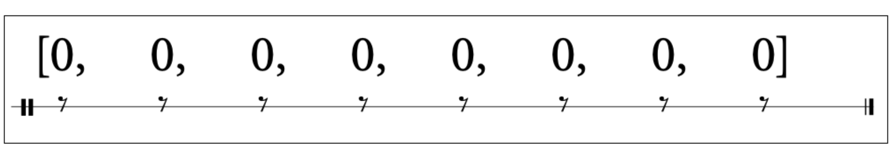

Figure 2. Array representing positions in time.

Each one of those components corresponds with a *division* of the space into its respective number (see Figure 3). So f_4—the fourth component—divides the span into 4 parts. Because 4 is a factor of 8, this might also be thought of as a division into "quarter-notes." The more onsets that coincide with this division, the higher the magnitude for the component. A steady quarter-note input has a maximal magnitude for a f_4 here.

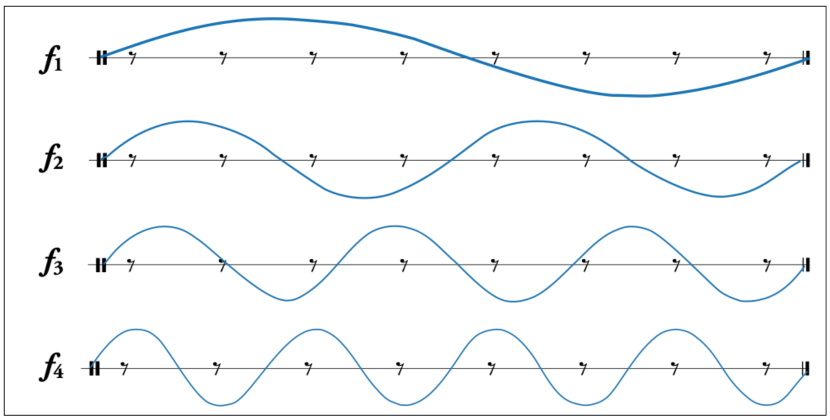

Figure 3. Division into respective components.

On a perceptual note, I argue that the DFT is theoretically similar to Edward Large's neural oscillator model—a model based around neurons firing together when we entrain (Figure 4a). Like the DFT, in Large's model as more onsets align with the same periodicity, the higher the magnitude for that periodicity. As an analogy, Large and Jones show a rolling circle symbolizing our attentive state (see Figure 4b).

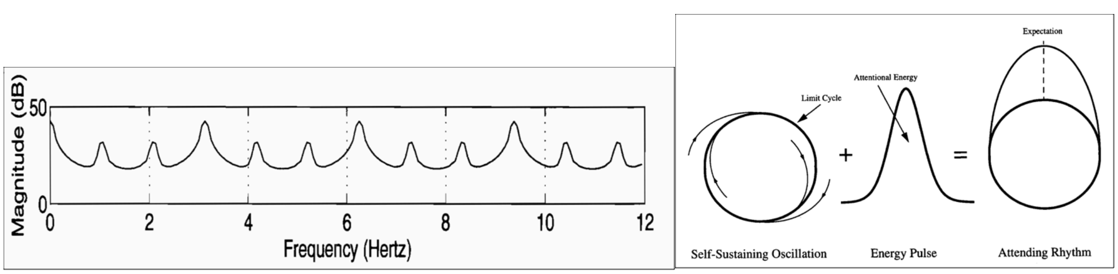

Figure 4a–b. Neural oscillator model (from Large 1994); Rolling circle analogy (from Large and Jones 1999).

We can directly map an "entrainment oscillator" to a DFT component with some simple trigonometry. So in a cognitively aware way of thinking, the DFT components are represented as a series of rolling circles at a certain subdivision. Each component, therefore, represents a particular *attentional state* (London 2012)**.** Throughout the presentation, I'll change between the various representations of DFT components—just know that they are all equal; the DFT components occupy an interesting theoretical space, conceptually sinusoids dividing the space, subdivisional projections in the music, or attentional states in entrainment—a three-faced coin.

For a DFT example, let's do a quick walkthrough on our favorite overly-theorized rhythm: the *tresillo* pattern—an *almost* triplet see (Figure 5). According to the DFT the rhythm is *maximally-even* distribution of 3 into 8 (see Figure 6). I call these visualizations *rhythmic profiles*. Each one of the components on the *x-axis* represents a division into that many parts: the 3rd component divides the measure into 3 parts—so we can consider the magnitude on the *y-axis* to represent a *goodness of fit*, or as how well the stimulus can be fit into a certain sinusoid.

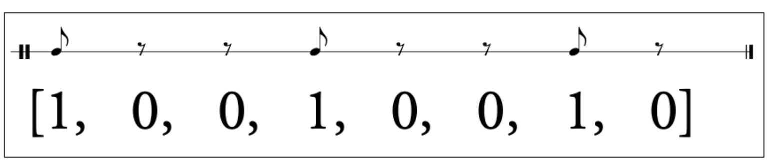

Figure 5. *Tresillo* timeline.

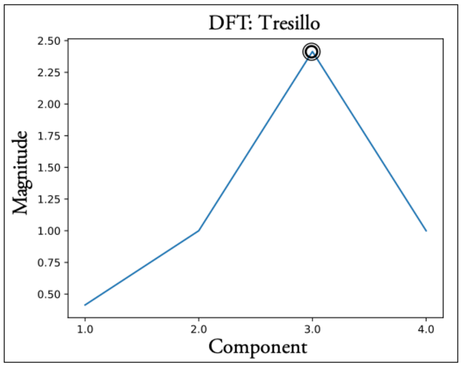

Figure 6. Rhythmic profile for a *tresillo*.

So that was the theoretical backing, but, just as important, we need to decide how to organize the rhythms we'll be examining. This is where the methodology is needed: the methodology is *how* we frame the musical object before analysis.

Part III. Methodology

We might say that the theory of this paper is formed epistemologically on connecting the DFT to cognitive theories of entrainment and that the interpretive results are guided by how the methodology encodes the musical object. For now, I'll just discuss the specific weighting system implemented here.

In this paper I've implemented a *rule-based system* based on Lerdahl and Jackendoff's *Metric Preference Rules* and the internal-clock accent rules from Povel and Essens (1985) (see Figure 7). The rules take a rhythm and systematically derives an *accent profile* from it. So using this unfamiliar Mozart example (see Figure 8)… we go through the rules and derive an accent profile.

ACCENT PROFILE RULES	
MPR 3: Event	+1 wherever there is a note
MPR 6: Bass Note	+1 to lowest note of any 5 contiguous units
PEC: Second of Two	+1 to second to two adjacent notes
PEC: Initial and Final Tones	+1 to both ends of an adjacent string of notes
Large Leaps	+1 to any leap of 5 semitones or more
Continuous/Contour	+1 to all changes in the contour: contour is established after 2 moves in the same direction
MPR 1: Parallelism	+1 to parallelism projects

PEC = Povel/Essens Clock (Rules)

MPR = Metric Preference Rules (Lerdahl and Jackendoff)

Figure 7. Accent profile Rules

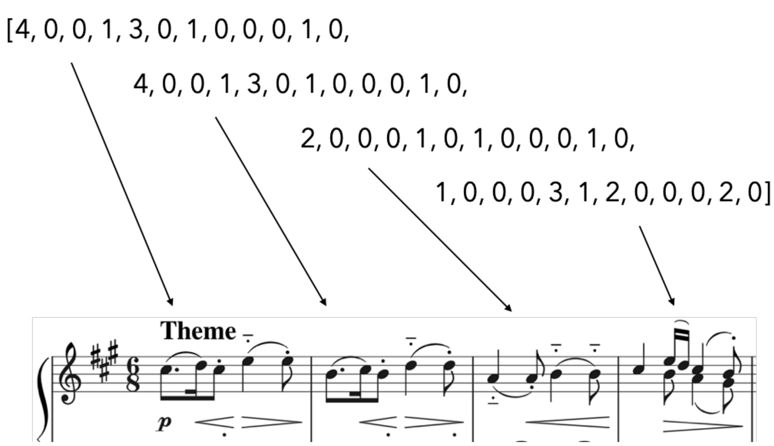

Figure 8. Accent profiles on melody of Mozart K. 331.

So, after going through each parameter, the rhythm is described in terms of an array that we can use the DFT on. Let's listen with it in mind (**https://tinyurl.com/UchidaMozart**). What I found interesting here, is that, while the general meter is conveyed merely through events, the weighting isn't a great representation of the Mozart example… I think these rules are good for deriving accent profiles for Math Rock—implying a difference between how we listen to different styles. Now let's put the methodology to use with an analysis American Football's Never Meant.

Part IV. Analytical Vignettes: Never Meant

The iconic beginning of "Never Meant," shown in Figure 9, starts with a little intro before 6 rim clicks lead into the lead guitar enters (**https://tinyurl.com/NeverMeantG1**). You may have noticed that in the accent profiles, I have intentionally left out drum patterns due to their clear paradigms to evoke certain meters, which is something *we might discuss in the Q and A* (in lieu of the Q+A, if you have questions I encourage the reader to email me with any questions or comments). Per the guitar, I hear the lowest notes in the pattern sticking out and, after calculating the accent profile, the array generally shows that too. So now we use the DFT on the array, and it results in the rhythmic profile in Figure 10. There are 4 peaks corresponding to divisions of the rhythm into 3, 6, 9, and 12 parts. Because components 3,6, and 12 are factors of the rhythm's 24-eighth-note length, they correspond to notational subdivisions. The high magnitude for these components suggest that this rhythm projects whole-, half-, and quarter-note pulses. The DFT profile is therefore implying 3 measures of 4/4.

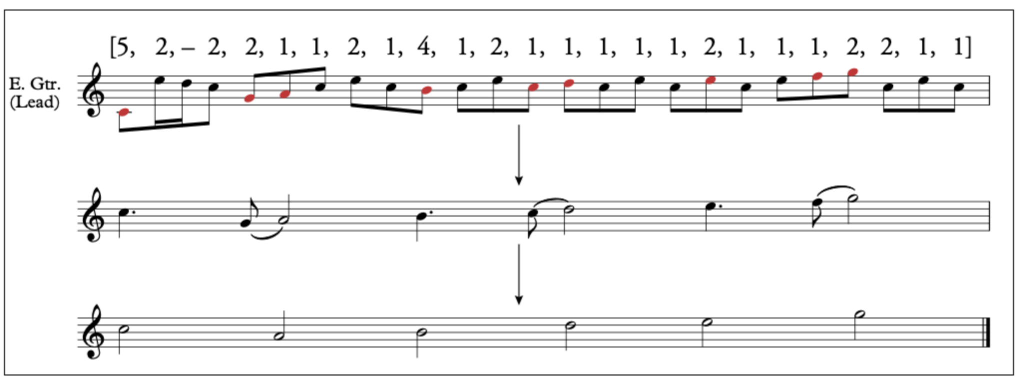

Figure 9. "Never Meant": guitar 1 and accent profile.

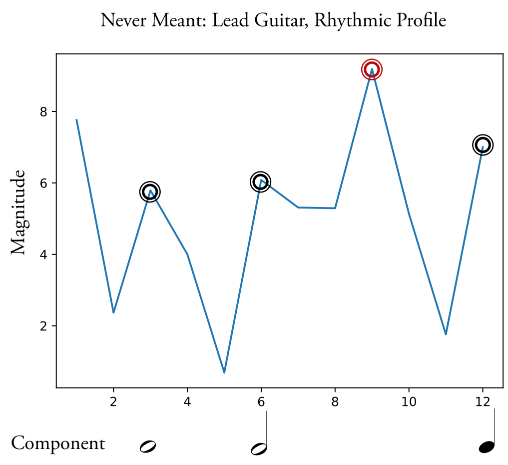

Figure 10. "Never Meant" rhythmic profile.

Returning to the song: after the pattern of the lead guitar has looped 4 times the second guitar enters and stacks another cycle on top spanning 24-eighth notes (Figure 11). Let's listen—pay particular attention to the lower line of the second guitar (**https://tinyurl.com/NeverMeantG2**). That lower line projecting 7+6+7+4 in eighth notes sticks out to me, and, again, the weighting does show that. In the rhythmic profile (Figure 12), there are peaks around components 4, 7, and 10/11. The fourth component is high because the projection 7+6+7+4 is *nearly* an even division into 4 parts.

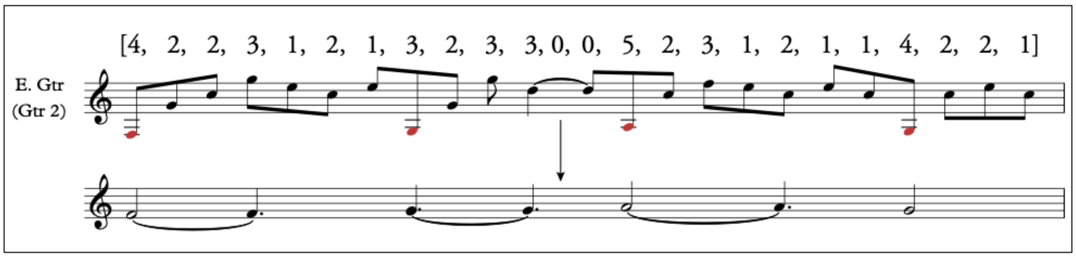

Figure 11. "Never Meant" guitar 2 and accent profile.

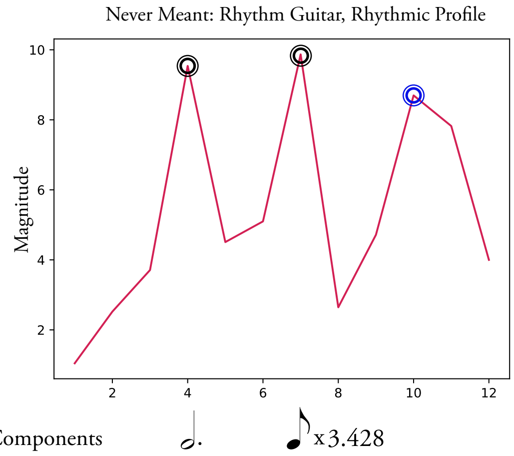

Figure 12. “Never Meant” rhythmic profile.

As shown on the clocks in Figure 13a and 13b, you can see that the lower line occupies positions {0, 7, 13, 20} which is very close to an even division: {1, 7, 13, 19}—only 2 onset shifts away. Let’s listen again and see if you can hear how that bass line is *nearly even*… Even if it’s offset a bit, the lower notes feel like they’re afterbeat syncopations (**https://tinyurl.com/NeverMeantG2**).

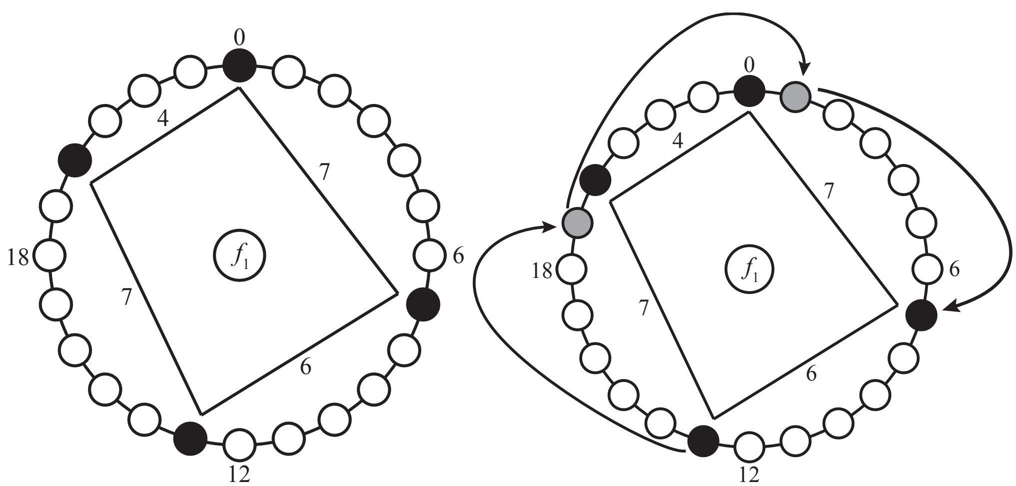
Figure 13a. Nearly even division of 4 (from the lower line); 13b. Transformation to even distribution.

The 10th component of the excerpt (see Figure 12) doesn’t correspond with an integer subdivision. It corresponds to “2.4” eight-note projections which… doesn’t musically make sense. Instead, we might interpret this as the approximate average between two projections: the quarter note and dotted quarter. In other words, competing subdivisions—a metric conflict.

To represent metric conflict, Rick Cohn introduces a visual which divides the levels of a meter into its duple and triple subdivisions—"metric ski-hill graphs" (Figure 14a). The ski-hill graph can be directly mapped onto DFT components (Figure 14b). Not only that, but because each component has *magnitude*, it can quantitatively show a depth to *how much* a subdivision or *conflict* is conveyed. In other words, it creates a topological surface with gradients and degrees, rather than a binary "present or not."

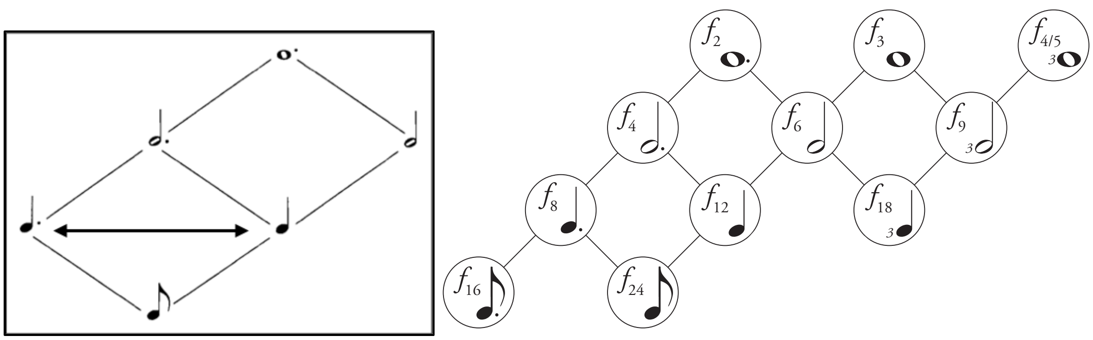

Figure 134a. Metric ski-hill graph; 14b. ski-hill graph with DFT components.

Reviewing Never Meant: the rhythm is a near even division into 4 even parts, and at a lower level it has metrical conflict between the quarter and dotted-quarter. Let's listen one more time and see if you can hear both of these phenomena. **(https://tinyurl.com/NeverMeantG2)**

Analytical Vignettes: Pool

In 2010, guitarist and vocalist Ikumi "Ikkyu" Nakajima formed the band Tricot. A 2015 issue of the *Rolling Stone*, entitled "10 New Artists You Need to Know" has described them as "adrenalized math rock." This is the beginning of "Pool" off of their album *T H E* **Bhttps://www.youtube.com/watch?v=TZjTXh_zaXc 0:00–:50**). After a short introduction, the guitar projects a grouping of 5 and repeats that grouping 3 times, making a cycle of 15-eighth notes (Figure 14). A DFT of the array yields the rhythmic profile in Figure 15. It picks up the 5-projections in f_3 and also shows a high magnitude for f_6 . f_6 corresponds to a further division of the $\frac{5}{8}$ into 2 parts, showing the inherent back-and-forth in the subdivisional conflict of a $\frac{5}{8}$ meter.

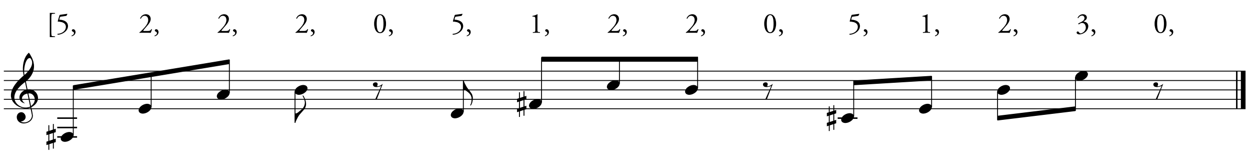

Figure 14. "Pool" guitar cycle.

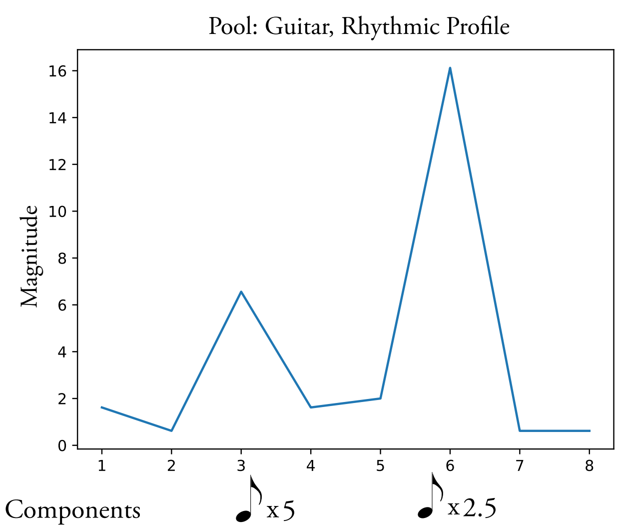

Figure 15. "Pool" rhythmic profile.

After the guitar, Ikkyu then enters with a voice part, repeating a grouping of 10 three times, making a cycle of 30 (Figure 16). The 10-projections are split into 3+3+2+2.[2] The rhythmic profile is shown in Figure 17. The DFT unsurprisingly picks up on the 10-projections

[2] Except for one absent onset towards the end.

in f_3 , and shows an underlying quarter note subdivision throughout, but, interestingly, there are peaks at f_9, and f_{12}.

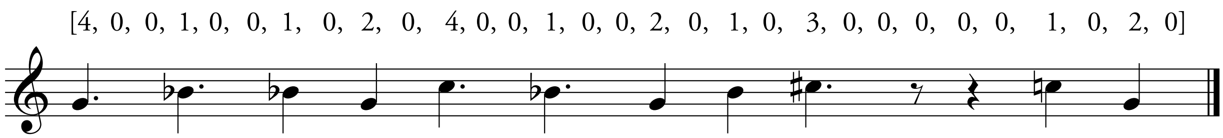

Figure 16. "Pool" vocal line.

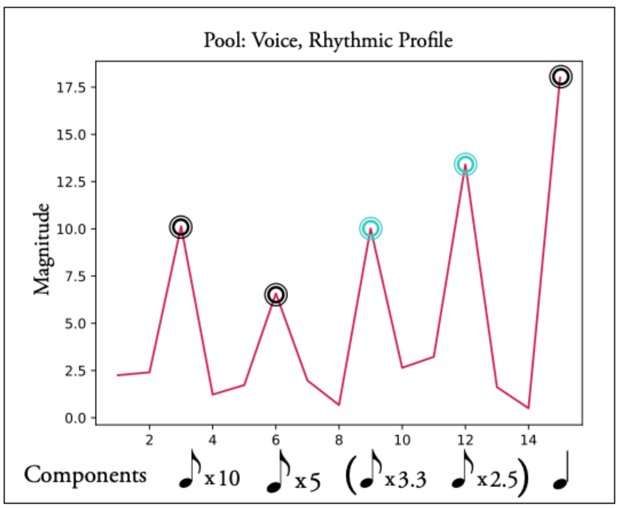

Figure 17. "Pool" voice line rhythmic profile

Both components 9 and 12 represent divisions of the $\frac{10}{8}$ measures into 3 and 4 parts respectively. This shows that the rhythm 3+3+2+2 is structurally *close* to a maximally even division into 4 parts—3+2+3+2 (Figure 18a)—and 3 parts—3+3+4 (see Figure 18b). This suggests, more obviously, that projections of 3 and 2 (and 4) are prominent, both vying for, or perhaps sharing the role of tactus. An isochronous interpretation of non-isochronous meters—like the DFT—therefore shows the average of the competing subdivision projections.

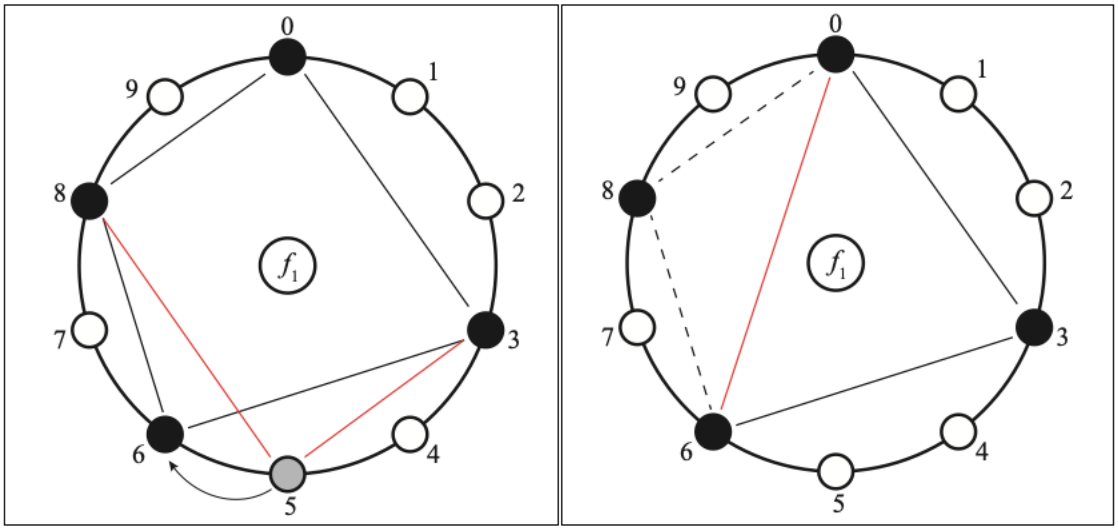

Figure 18a. Nearly even division into 4 parts; and 18b. nearly even division into 3 parts

Analytical Vignettes: Cat Fantastic

Finally, we turn to TTNG's "Cat Fantastic." Examples thus far have been strict ostinato; let's look at an example which complicates that. The bridge starts with a solo electric bass, and is later joined by a melody doubled in the voice and guitar (see Figure 19) (**https://tinyurl.com/CatFantastic**). The bridge repeats 3 patterns of $\frac{7}{8}$ followed by a measure that changes with each iteration. For every repetition, the altered measured adds an extra eighth note, moving from $\frac{3}{8}$, to $\frac{4}{8}$, to $\frac{5}{8}$, and, finally, to $\frac{6}{8}$. I call this a *Milankovitch cycle,* named after the astronomer who first described orbital cycles that change over time; just like the planets, this cycle changes ever so slightly with each rotation.

Figure 19. “Cat Fantastic” Milankovitch cycle.

Figure 20 shows the rhythmic profiles of each iteration in the Milankovitch cycle calculated with the bass and voice line. Despite the slight variations between each iteration, each profile retains the general shape. They have peaks that roughly represent the $\frac{7}{8}$ measure level, an asymmetrical division of that into two parts, and a further division. In this way, the profiles resemble our common metric hierarchies.

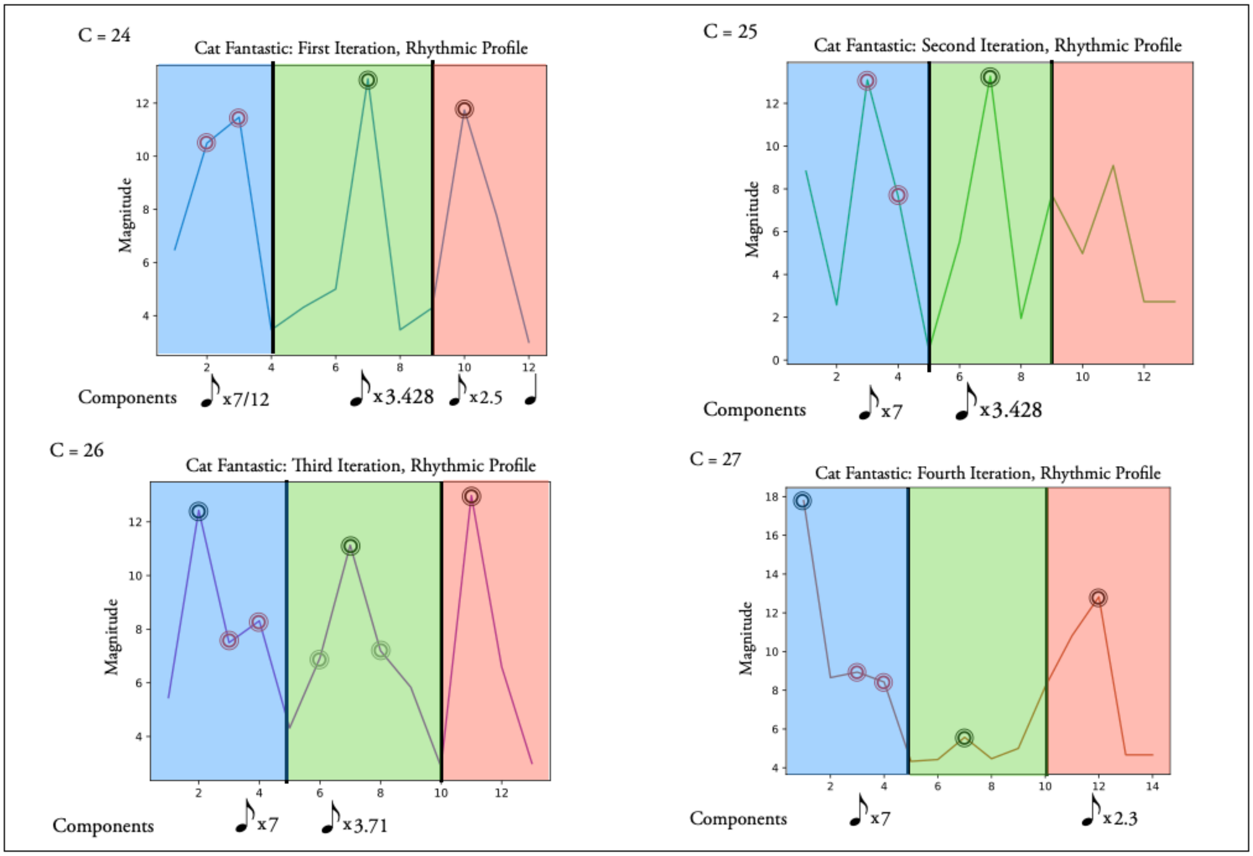

Figure 20. "Cat Fantastic" rhythmic profiles where C is the cycles length in eighth notes.

The rhythmic patterns in the first and third iteration are essentially identical and their rhythmic profiles show that too (Figure 20). There is one significant change in the profiles though. Due to the extension of the extra measure, the pattern gradually develops a larger-scale symmetry. Notice, as the cycle continues to grow, the pattern approaches a larger 2- and 4- part symmetry (Figure 21). So in the smaller cycle (C=24), meter and hypermeter are lumped together, but in a larger cycle (C=26), the meter and hypermeter becomes untangled. I think this feature captures the phenomenological experience of the progression's process: the $\frac{3}{8}$ slowly snails out, and, as it stretches, it alters the symmetry of the cycle. Let's listen once more, focusing on this sprawling symmetry (**https://tinyurl.com/CatFantasticCut**).

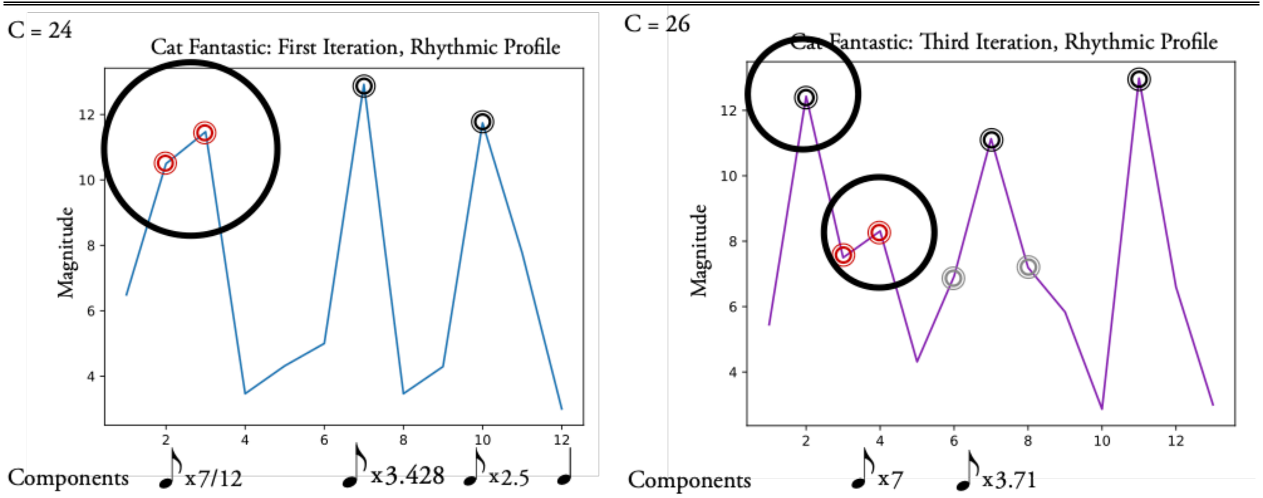

Figure 21. "Cat Fantastic" sprawling symmetry.

Conclusion

My argument situates the DFT as a mathematical equation appropriate for modeling a cognitive experience of meter, and does so on underexamined music popular today. I've shown 3 math rock examples: "Never Meant" which has metric conflict within a standard meter, "Pool" which has metric conflict and near-evenness despite asymmetrical meters, and "Cat Fantastic" a piece with changing cycles and asymmetrical meters—and, hopefully, I've shown how the DFT captures all of these features. While I've demonstrated how both Cohn's metric-ski hills and Large's oscillator models can be adopted and unified by DFT components, the field of metric theory is vast—this was only a *Fourier* into its troughs.